17216

16172

3 Hours / 100 Marks Seat No. | | | | | | | | |

Instructions – (1) All Questions are *Compulsory*.

(2) Answer each next main Question on a new page.

(3) Figures to the right indicate full marks.

(4) Use of Non-programmable Electronic Pocket Calculator is permissible.

(5) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

Marks

1. Solve any TEN of the following: **20**

a) Find the value of: $i^{20} + i^{30} + i^{40} + i^{50}$

b) Express: $(2 + 3i)(1 - 4i)$ in the form $a + ib$

c) Find 'a' if $f(x) = ax + 10$ and $f(1) = 13$.

d) Define: Even and odd function.

e) Evaluate: $\lim_{x \to 3} \frac{\sqrt{x} + \sqrt{3}}{x + 3}$

f) Evaluate: $\lim_{x \to 0} x.\operatorname{cosec} x$

g) Evaluate: $\lim_{x \to 0} \frac{a^x + b^x - 2}{x}$

h) Evaluate: $\lim_{x \to 0} \frac{\log(1 + 5x)}{x}$

i) If $y = 2e^{3x} + \tan x - \cos 2x + 9\sin^{-1} x$, find $\frac{dy}{dx}$.

P.T.O.

Marks

j) If $y = \frac{\log x}{x}$, find $\frac{dy}{dx}$.

k) Differentiate $7x^5 - 11x^2$ w.r.t. $7x^2 - 15x$.

l) Differentiate w.r.t. x : $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$

m) Prove that the root of the equation $x^3 - x - 4 = 0$ lies between 0 and 2.

n) Find the first iteration by using Jacobi's method for the following system of equations:

$10x + y + 2z = 13,\ 3x + 10y + z = 14,\ 2x + 3y + 10z = 15$

2. Solve any FOUR of the following: **16**

a) If $f(x) = ax^2 + bx + 2$ and $f(1) = 3, f(4) = 42$, find a and b.

b) If $f(x) = \frac{2x+3}{3x-2}$, Prove that $f\left[f(x)\right] = x$

c) Separate into real and imaginary parts of:

$$\frac{2+i}{(3-i)(1+2i)}$$

d) Solve: $(4-5i)x + (2+3i)y = 10 - 7i$

e) Simplify: $\frac{(\cos 3\theta + i\sin 3\theta)^4 (\cos 4\theta - i\sin 4\theta)^5}{(\cos 4\theta + i\sin 4\theta)^3 (\cos 5\theta + i\sin 5\theta)^{-4}}$

f) Find all the cube roots of (–1)

3. Solve any FOUR of the following: **16**

a) If $f(x) = \log[1 + \tan x]$, show that $f\left(\pi/4 - x\right) = \log 2 - f(x)$.

b) If $f(x) = x^2 - 3x + 4$ then solve: $f(1-x) = f(2x+1)$

c) Evaluate: $\lim_{x \to 5} \frac{x^2 - 9x + 20}{x^2 - 6x + 5}$

d) Evaluate: $\lim_{x \to 3} \frac{\sqrt{x^2+1} - \sqrt{10}}{x - 3}$

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e) Evaluate: $\lim_{x \to a} \frac{\sin x - \sin a}{x - a}$

f) Evaluate: $\lim_{x \to 0} \frac{15^x - 5^x - 3^x + 1}{x . \sin x}$

4. Solve any FOUR of the following: **16**

a) Differentiate w.r.t. $x : x^{\sin 2x}$

b) If $x = 3\cos\theta - \cos 3\theta,\ y = 3\sin\theta - \sin 3\theta$ then find dy/dx.

c) Differentiate w.r.t. x $(\tan x)^x$.

d) Differentiate $x^{\sin^{-1}x}$ w.r.t. $\sin^{-1}x$.

e) If $xy = \log(xy)$ show that $\frac{dy}{dx} = -\frac{y}{x}$

f) If U and V are differentiable functions of x and $y = u + v$ then prove that: $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$.

5. Solve any FOUR of the following: **16**

a) Evaluate: $\lim_{x \to 0} \frac{\log(2 + x) - \log(2 - x)}{x}$

b) Show that the roots of the equation $x^3 - 9x + 1 = 0$ lies between 2 and 3. Obtain the roots by Bisection method. (3 iterations only)

c) Using Newton–Raphson method, Evaluate: $\sqrt[3]{100}$ (Upto three iterations only)

d) Using Regula – Falsi method, find the root of $xe^x - 3 = 0$ (three iterations only)

e) Using Bisection method, find the approximate root of $x^3 - 2x - 5 = 0$ in the interval (2, 3) (3 iterations only)

f) Find the roots of the equation using Newton–Raphson method $x^2 - 4x - 6 = 0$ near to 5. (three iterations only)

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6. Solve any FOUR of the following: **16**

a) Solve by Gauss Elimination method:
$x + y + z = 6,\ 3x - y + 3z = 10,\ 5x + 5y - 4z = 3$

b) By using first principle, prove that:
$\frac{d}{dx}(\sin x) = \cos x.$

c) Solve by Jacobi's method upto 3 iterations only:
$30x + y + z = 32,\ x + 30y + z = 32,\ x + y + 30z = 32$

d) Solve by Gauss–seidal method (3 iterations only)
$6x + y + z = 105,\ 4x + 8y + 3z = 155,\ 5x + 4y - 10z = 65$

e) Solve by Gauss Elimination method:
$x + y + z = 4,\ 2x + y + z = 5,\ 3x + 2y + z = 7$

f) Solve by Jacobi's method:
$4x + y + 2z = 12,\ -x + 11y + 4z = 33,\ 2x - 3y + 8z = 20$
(3 iterations only)